

Exercise 15

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$y = \int_1^{3x+2} \frac{t}{1+t^3} dt$$

Solution

According to part 1 of the fundamental theorem of calculus,

$$\frac{d}{dx} \int_a^x f(t) dt = f(x).$$

In order to make the upper limit a single variable, let $u = 3x + 2$.

$$y = \int_1^u \frac{t}{1+t^3} dt$$

As a result, using the chain rule,

$$\begin{aligned} y' &= \frac{d}{dx} \int_1^u \frac{t}{1+t^3} dt \\ &= \frac{du}{dx} \frac{d}{du} \int_1^u \frac{t}{1+t^3} dt \\ &= \frac{du}{dx} \left(\frac{u}{1+u^3} \right) \\ &= 3 \left[\frac{(3x+2)}{1+(3x+2)^3} \right] \\ &= \frac{3(3x+2)}{1+(3x+2)^3}. \end{aligned}$$